MATH 170 – CHAPTER 3 Name: .

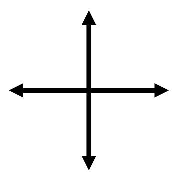
3.1 Reference Angle

Need To Know

- Reference Angle
  - Definitions
  - Formulas
- Exact Values

Reference Angle solves dilemma

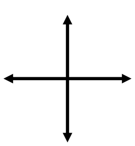
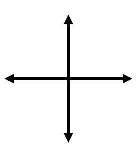
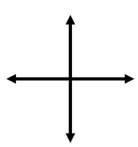
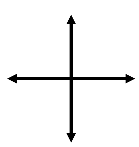
Find θ in standard position with a terminal side through $(-1, -\sqrt{3})$.



Reference Angle Definition

The *reference angle* for θ in standard position is the positive acute angle between the terminal side of θ and the x-axis. Denoted: the reference angle θ is $\hat{\theta}$.

$\theta = 45°$ $\theta = 150°$ $\theta = 240°$ $\theta = 315°$

IF $\theta \in$ QI $\theta \in$ QII $\theta \in$ QIII $\theta \in$ QIV

Reference Angle & Exact Values

Find the reference angle for each angle.

$\theta = 97.5°$　　　　　　$\theta = 1000°$

<u>Reference Angle Property</u>

The trig function of an angle is

____

(except for_______________

which you decide based on which quadrant θ terminates).

Exact Values - Practice

<u>Reference Angle Property</u>

trig (θ) = _________________

Find the exact values of each:

$\sin 225° =$

$\cos 330° =$

$\csc 300° =$

Exact Values - Practice

Find θ between 0° and 360° if

$\cos\theta = \frac{1}{\sqrt{2}}$ with θ in QII

Exact Values - Practice

Find θ between 0° and 360° if

$\sin \theta = -0.3090$ with θ in QIII

Exact Values - Practice

Find θ between 0° and 360° if

$\cot \theta = -0.1234$ with θ in QIV

end

3.2 Radian Measure

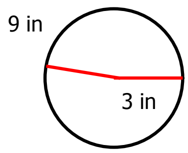
<u>Need To Know</u>

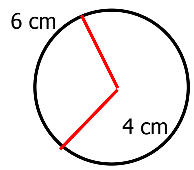
- Two types of measure
  - Definition of radian
  - Formula for radian measure
- Converting between Degrees and Radians
- Exact Values

Radian Measure

Definition:
In a circle, a central angle that cuts off an ____________
equal to the ___________ is an angle measure of 1 radian.

Definition:
For angle θ, in a circle of radius r cuts an arc length of s,
then the measure of θ in radians is ______





Conversions: deg ←→ rad

$360° =$ ______

Conversions: deg ←→ rad

a) Draw angle,
b) Find the reference angel
c) Convert both to radians

$\theta = -120°$

a) Draw angle,
b) Find the reference angel
c) Convert both to degrees

$\theta = 7\pi/12$

Exact Values

Memorize the basic conversions

Deg	Rad
$0°$	
$30°$	
$45°$	
$60°$	
$90°$	

Evaluate:

$2\cos\left(\frac{\pi}{6}\right)$ $\qquad$ $\sin\left(3\cdot\frac{\pi}{6}\right)$

Exact Values

Evaluate:

$\cos\left(\frac{4\pi}{3}\right)$

$\csc\left(\frac{7\pi}{6}\right)$

$4\tan\left(-\frac{\pi}{4}\right)$

end

3.3 Trig Definitions - Circle

<u>Need To Know</u>

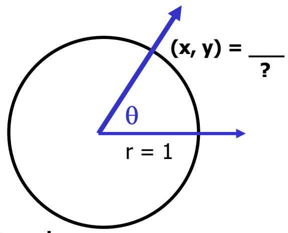
- Circle Definitions
- Calculator examples
- Domain and Range

Unit Circle Definitions

Goal: See old trig functions in a new way.

Recall: ___________________

Conclusion:



cos(θ) = ___________

sin(θ) = ___________

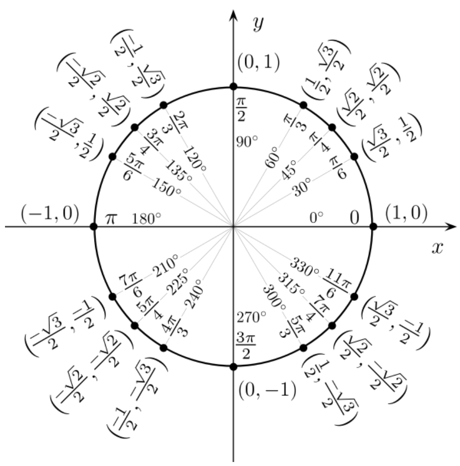
where (x, y) is the point where θ intersects the unit circle.

Unit Circle

It is good for seeing relationships **BUT <u>NOT</u>** to be memorized.

Example:
Find all values of θ in radians where

$$\sin\theta = -\frac{1}{\sqrt{2}}$$



Practice

Find all values of θ between 0 and 2π radians:

$$\cos\theta = \frac{\sqrt{3}}{2} \qquad\qquad \sin\theta = -\frac{1}{\sqrt{2}}$$

Practice

If t is the arc distance from (1,0) to (-0.9422, 0.3350) on the unit circle, find sin t, cos t and tan t.

Calculator Practice

Evaluate each:

$\cos\frac{\pi}{4}$ $\qquad\qquad$ $\sin\frac{\pi}{7}$

Find θ in radians if $\sin\theta = 0.8$

Domain and Range

Recall –

The input to a function is called the ____________.

The output of a function is called the ___________.

A function pairs each domain with only one range.

Domain – can be t, as a real number, or θ in radians

sin t, cos t: ________________________.

tan t, sec t: All real numbers except $t = \pi/2 + k\pi$ for any k

cot t, csc t: All real numbers except $t = k\pi$ for any k

Range

sin t, cos t: ____________.

tan t, cot t: All real numbers, $(-\infty, \infty)$

sec t, csc t: $(-\infty, -1]$ or $[1, \infty)$

end

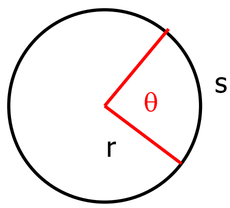
3.4 Arc Length & Sector Area

<u>Need To Know</u>

- Arc Length formula
- Sector Area formula
- Read 4.1 to get a head start
- Be sure to bring calculator everyday now

Recall Radian Measure

Recall:



Arc Length Formula:

____

Application

How far does a pendulum travel from side to side?
It swings 20° in 1 sec and it is 4 feet long.

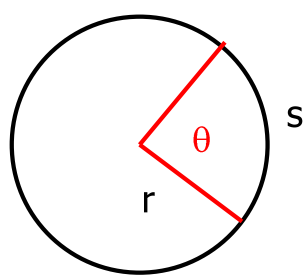
Application

Plan? 1) find θ 2) proportions 3) find t

How long will it take the space shuttle to travel 8400 miles? It is 200 miles up and orbits the earth every 6 hours. ($r_{earth} \simeq 4000$ miles)

Area of a sector

Set A = the area of the sector made by θ.
Consider proportions.



Sector Area Formula:

______________________________________.

Application

Find the area of a sector if $\theta = \pi/4$ and r = 4 in.

Application

A lawn sprinkler sprays out 30 ft and rotates 60°
What is the area it covers?

end